Elementary Derivation of the Dirac Equation. IX

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The transversality in the electrodynamic hydrogen field is discussed.

In [1] the attempt is made to ensure the obligatory transversality of the electrodynamic hydrogen field through the requirement that the inner product of the field with centrally directed vectors should vanish because of its central symmetry. Another problem there was the establishment of the field components having the same phase velocity.

In contrast to this we will here attempt to ensure the transversality from the beginning through a suitable ansatz, and to obtain the equality of the velocities of the field components by completely exhausting the full potential of the solutions.

As is well known, covariant Dirac-like electrodynamics

$$\begin{aligned} &\operatorname{rot} \boldsymbol{H} = \frac{\varepsilon}{c}\dot{\boldsymbol{E}}, \\ &\operatorname{rot} \boldsymbol{E} = -\frac{\mu}{c}\dot{\boldsymbol{H}}, \\ &\operatorname{div} \boldsymbol{E} = 0, \qquad (1)\\ &\operatorname{div} \boldsymbol{H} = 0, \\ &\boldsymbol{E} \perp \operatorname{grad} \varepsilon, \quad \boldsymbol{H} \perp \operatorname{grad} \mu, \end{aligned}$$

can, according to Da Silveira [2], with the help of the relation

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}\cdot\boldsymbol{A}) = \boldsymbol{\nabla}\cdot\boldsymbol{A} + i\,\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A}), \qquad (2)$$

be brought into the form

$$\left[\begin{pmatrix} \mathbb{0} & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & \mathbb{0} \end{pmatrix}\cdot\boldsymbol{\nabla} - \begin{pmatrix} \varepsilon\,\mathbb{1} & \mathbb{0} \\ \mathbb{0} & \mu\,\mathbb{1} \end{pmatrix}\frac{\partial}{\partial(ct)}\right]\begin{bmatrix}\begin{pmatrix} i(\boldsymbol{\sigma}\cdot\boldsymbol{E}) \\ (\boldsymbol{\sigma}\cdot\boldsymbol{H}) \end{pmatrix}\end{bmatrix} = 0, \qquad (3)$$
$$\boldsymbol{E} \perp \operatorname{grad} \varepsilon, \quad \boldsymbol{H} \perp \operatorname{grad} \mu.$$

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Then (3), or (1) respectively, has, according to [3] (6), the two complex solutions

$$\Psi = [i\check{E}_3,\, i(\check{E}_1 + i\check{E}_2),\, \check{H}_3,\, (\check{H}_1 + i\check{H}_2)],$$
$$\check{\boldsymbol{E}} \perp \operatorname{grad} \varepsilon, \quad \check{\boldsymbol{H}} \perp \operatorname{grad} \mu; \qquad (4)$$
$$\Psi = [i(\hat{E}_1 - i\hat{E}_2),\, -i\hat{E}_3,\, (\hat{H}_1 - i\hat{H}_2),\, -\hat{H}_3],$$
$$\hat{\boldsymbol{E}} \perp \operatorname{grad} \varepsilon, \quad \hat{\boldsymbol{H}} \perp \operatorname{grad} \mu. \qquad (5)$$

The relations (4) and (5) can be rewritten as four systems of equations, an example of which is the system for the $\check{\boldsymbol{E}}$-field

$$\Psi_1 = i\check{E}_3, \quad \Psi_2 = i\check{E}_1 - \check{E}_2, \quad D_{\check{E}} = i\,e^{i\varphi},$$
$$0 = \check{E}_1\cos\varphi + \check{E}_2\sin\varphi + \check{E}_3\cot\vartheta. \qquad (6)$$

Analogous systems are obtained for the three remaining fields $\check{\boldsymbol{H}}$, $\hat{\boldsymbol{E}}$ and $\hat{\boldsymbol{H}}$. The solution of these four systems yields the electromagnetic fields $(\check{\boldsymbol{E}}, \check{\boldsymbol{H}})$ and $(\hat{\boldsymbol{E}}, \hat{\boldsymbol{H}})$ as follows:

$$\check{\boldsymbol{E}} = \left[\frac{+\Psi_1\cot\vartheta + i\Psi_2\sin\varphi}{D_{\check{E}}},\ \frac{+i\Psi_1\cot\vartheta - i\Psi_2\cos\varphi}{D_{\check{E}}},\ -i\Psi_1\right], \qquad (7)$$

$$\check{\boldsymbol{H}} = \left[\frac{i\Psi_3\cot\vartheta + \Psi_4\sin\varphi}{D_{\check{H}}},\ \frac{-\Psi_3\cot\vartheta - \Psi_4\cos\varphi}{D_{\check{H}}},\ \Psi_3\right];$$

and

$$\hat{\boldsymbol{E}} = \left[\frac{i\Psi_1\sin\varphi - \Psi_2\cot\vartheta}{D_{\hat{E}}},\ \frac{-i\Psi_1\cos\varphi + i\Psi_2\cot\vartheta}{D_{\hat{E}}},\ i\Psi_2\right],$$

$$\hat{\boldsymbol{H}} = \left[\frac{\Psi_3\sin\varphi + i\Psi_4\cot\vartheta}{D_{\hat{H}}},\ \frac{\Psi_3\cos\varphi + \Psi_4\cot\vartheta}{D_{\hat{H}}},\ -\Psi_4\right]. \qquad (8)$$

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Since Ψ is determined up to a complex constant factor only, the general solution is of the form

$$\boldsymbol{E} = \check{g}\,\check{\boldsymbol{E}} + \hat{g}\,\hat{\boldsymbol{E}} \quad \text{and} \quad \boldsymbol{H} = \check{g}\,\check{\boldsymbol{H}} + \hat{g}\,\hat{\boldsymbol{H}} \tag{9}$$

($\check{g}, \hat{g}$: arbitrarily complex).

$\boldsymbol{E}$ and $\boldsymbol{H}$ are now necessarily transverse and possess the hydrogen frequencies if ε and μ are specialized to the hydrogen values according to [4] (12). The solutions consist of aggregates which in turn are each composed of an even-numbered and an odd-numbered component of Ψ. This means that, in each component of $\boldsymbol{E}$ and $\boldsymbol{H}$, amplitudes with the azimuthal angular functions $e^{im\varphi}$ and $e^{i(m+1)\varphi}$ are brought together. Here not only the equality of the velocity of the components suggests itself, but also that the quantum number m be half-integer.

[1] H. Sallhofer, Z. Naturforsch. **39 a,** 692 (1984).
[2] Da Silveira, Z. Naturforsch. **34 a,** 646 (1979).
[3] H. Sallhofer, Z. Naturforsch. **40 a,** 94 (1985).
[4] H. Sallhofer, Z. Naturforsch. **33 a,** 1378 (1978).

Corrigendum zu [3]:

The superscript k in Ψ^k is to be replaced by h (for hermitian conjugate) everywhere, yielding Ψ^h. In (4), (5), (7), (8), (10), and (15) replace $\gamma^{(k)}$ by $\gamma^{(4)}$, γ_k by γ_4, and ∂_k by ∂_4. The right hand side of (6) is to be understood as a matrix also.

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