

Elementary Derivation of the Dirac Equation. IX

Hans Sallhofer

Z. Naturforsch. **40a**, 651–652 (1985);
received May 9, 1985

The transversality in the electrodynamic hydrogen field is discussed.

In [1] the attempt is made to ensure the obligatory transversality of the electrodynamic hydrogen field through the requirement that the inner product of the field with centrally directed vectors should vanish because of its central symmetry. Another problem there was the establishment of the field components having the same phase velocity.

In contrast to this we will here attempt to ensure the transversality from the beginning through a suitable ansatz, and to obtain the equality of the velocities of the field components by completely exhausting the full potential of the solutions.

As is well known, covariant Dirac-like electrodynamics

$$\begin{aligned} \operatorname{rot} \mathbf{H} &= \frac{\varepsilon}{c} \dot{\mathbf{E}}, \\ \operatorname{rot} \mathbf{E} &= -\frac{\mu}{c} \dot{\mathbf{H}}, \\ \operatorname{div} \mathbf{E} &= 0, \\ \operatorname{div} \mathbf{H} &= 0, \\ \mathbf{E} &\perp \operatorname{grad} \varepsilon, \quad \mathbf{H} \perp \operatorname{grad} \mu, \end{aligned} \quad (1)$$

can, according to Da Silveira [2], with the help of the relation

$$(\boldsymbol{\sigma} \cdot \nabla)(\boldsymbol{\sigma} \cdot \mathbf{A}) = \nabla \cdot \mathbf{A} + i \boldsymbol{\sigma} \cdot (\nabla \times \mathbf{A}), \quad (2)$$

be brought into the form

$$\left[\begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \cdot \nabla - \begin{pmatrix} \varepsilon \mathbf{1} & 0 \\ 0 & \mu \mathbf{1} \end{pmatrix} \frac{\partial}{\partial (ct)} \right] \begin{bmatrix} i(\boldsymbol{\sigma} \cdot \mathbf{E}) \\ (\boldsymbol{\sigma} \cdot \mathbf{H}) \end{bmatrix} = 0, \quad (3)$$

$$\mathbf{E} \perp \operatorname{grad} \varepsilon, \quad \mathbf{H} \perp \operatorname{grad} \mu.$$

Then (3), or (1) respectively, has, according to [3] (6), the two complex solutions

$$\begin{aligned} \Psi &= [i\tilde{E}_3, i(\tilde{E}_1 + i\tilde{E}_2), \tilde{H}_3, (\tilde{H}_1 + i\tilde{H}_2)], \\ \tilde{\mathbf{E}} &\perp \operatorname{grad} \varepsilon, \quad \tilde{\mathbf{H}} \perp \operatorname{grad} \mu; \end{aligned} \quad (4)$$

$$\begin{aligned} \Psi &= [i(\hat{E}_1 - i\hat{E}_2), -i\hat{E}_3, (\hat{H}_1 - i\hat{H}_2), -\hat{H}_3], \\ \hat{\mathbf{E}} &\perp \operatorname{grad} \varepsilon, \quad \hat{\mathbf{H}} \perp \operatorname{grad} \mu. \end{aligned} \quad (5)$$

The relations (4) and (5) can be rewritten as four systems of equations, an example of which is the system for the $\tilde{\mathbf{E}}$ -field

$$\begin{aligned} \Psi_1 &= i\tilde{E}_3, \quad \Psi_2 = i\tilde{E}_1 - \tilde{E}_2, \quad D_{\tilde{\mathbf{E}}} = i e^{i\varphi}, \\ 0 &= \tilde{E}_1 \cos \varphi + \tilde{E}_2 \sin \varphi + \tilde{E}_3 \cot \vartheta. \end{aligned} \quad (6)$$

Analogous systems are obtained for the three remaining fields $\tilde{\mathbf{H}}$, $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$. The solution of these four systems yields the electromagnetic fields $(\tilde{\mathbf{E}}, \tilde{\mathbf{H}})$ and $(\hat{\mathbf{E}}, \hat{\mathbf{H}})$ as follows:

$$\tilde{\mathbf{E}} = \left[\frac{+\Psi_1 \cot \vartheta + i\Psi_2 \sin \varphi}{D_{\tilde{\mathbf{E}}}}, \frac{+i\Psi_1 \cot \vartheta - i\Psi_2 \cos \varphi}{D_{\tilde{\mathbf{E}}}}, -i\Psi_1 \right], \quad (7)$$

$$\tilde{\mathbf{H}} = \left[\frac{i\Psi_3 \cot \vartheta + \Psi_4 \sin \varphi}{D_{\tilde{\mathbf{H}}}}, \frac{-\Psi_3 \cot \vartheta - \Psi_4 \cos \varphi}{D_{\tilde{\mathbf{H}}}}, \Psi_3 \right];$$

and

$$\hat{\mathbf{E}} = \left[\frac{i\Psi_1 \sin \varphi - \Psi_2 \cot \vartheta}{D_{\hat{\mathbf{E}}}}, \frac{-i\Psi_1 \cos \varphi + i\Psi_2 \cot \vartheta}{D_{\hat{\mathbf{E}}}}, i\Psi_2 \right],$$

$$\hat{\mathbf{H}} = \left[\frac{\Psi_3 \sin \varphi + i\Psi_4 \cot \vartheta}{D_{\hat{\mathbf{H}}}}, \frac{\Psi_3 \cos \varphi + \Psi_4 \cot \vartheta}{D_{\hat{\mathbf{H}}}}, -\Psi_4 \right]. \quad (8)$$

Reprint requests to Dr. H. Sallhofer, Fischerstrasse 12, A-5280 Braunau, Austria.

0340-4811 / 85 / 0600-0651 \$ 01.30/0. – Please order a reprint rather than making your own copy.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition “no derivative works”). This is to allow reuse in the area of future scientific usage.

Since Ψ is determined up to a complex constant factor only, the general solution is of the form

$$\mathbf{E} = \check{g} \check{\mathbf{E}} + \hat{g} \hat{\mathbf{E}} \quad \text{and} \quad \mathbf{H} = \check{g} \check{\mathbf{H}} + \hat{g} \hat{\mathbf{H}} \quad (9)$$

($\check{g}, \hat{g}$: arbitrarily complex).

$\mathbf{E}$ and $\mathbf{H}$ are now necessarily transverse and possess the hydrogen frequencies if ε and μ are specialized to the hydrogen values according to [4] (12). The

solutions consist of aggregates which in turn are each composed of an even-numbered and an odd-numbered component of Ψ . This means that, in each component of $\mathbf{E}$ and $\mathbf{H}$, amplitudes with the azimuthal angular functions $e^{im\varphi}$ and $e^{i(m+1)\varphi}$ are brought together. Here not only the equality of the velocity of the components suggests itself, but also that the quantum number m be half-integer.

- [1] H. Sallhofer, Z. Naturforsch. **39a**, 692 (1984).
- [2] Da Silveira, Z. Naturforsch. **34a**, 646 (1979).
- [3] H. Sallhofer, Z. Naturforsch. **40a**, 94 (1985).
- [4] H. Sallhofer, Z. Naturforsch. **33a**, 1378 (1978).

Corrigendum zu [3]:

The superscript k in Ψ^k is to be replaced by h (for hermitian conjugate) everywhere, yielding Ψ^h . In (4), (5), (7), (8), (10), and (15) replace $\gamma^{(k)}$ by $\gamma^{(4)}$, γ_k by γ_4 , and ∂_k by ∂_4 . The right hand side of (6) is to be understood as a matrix also.